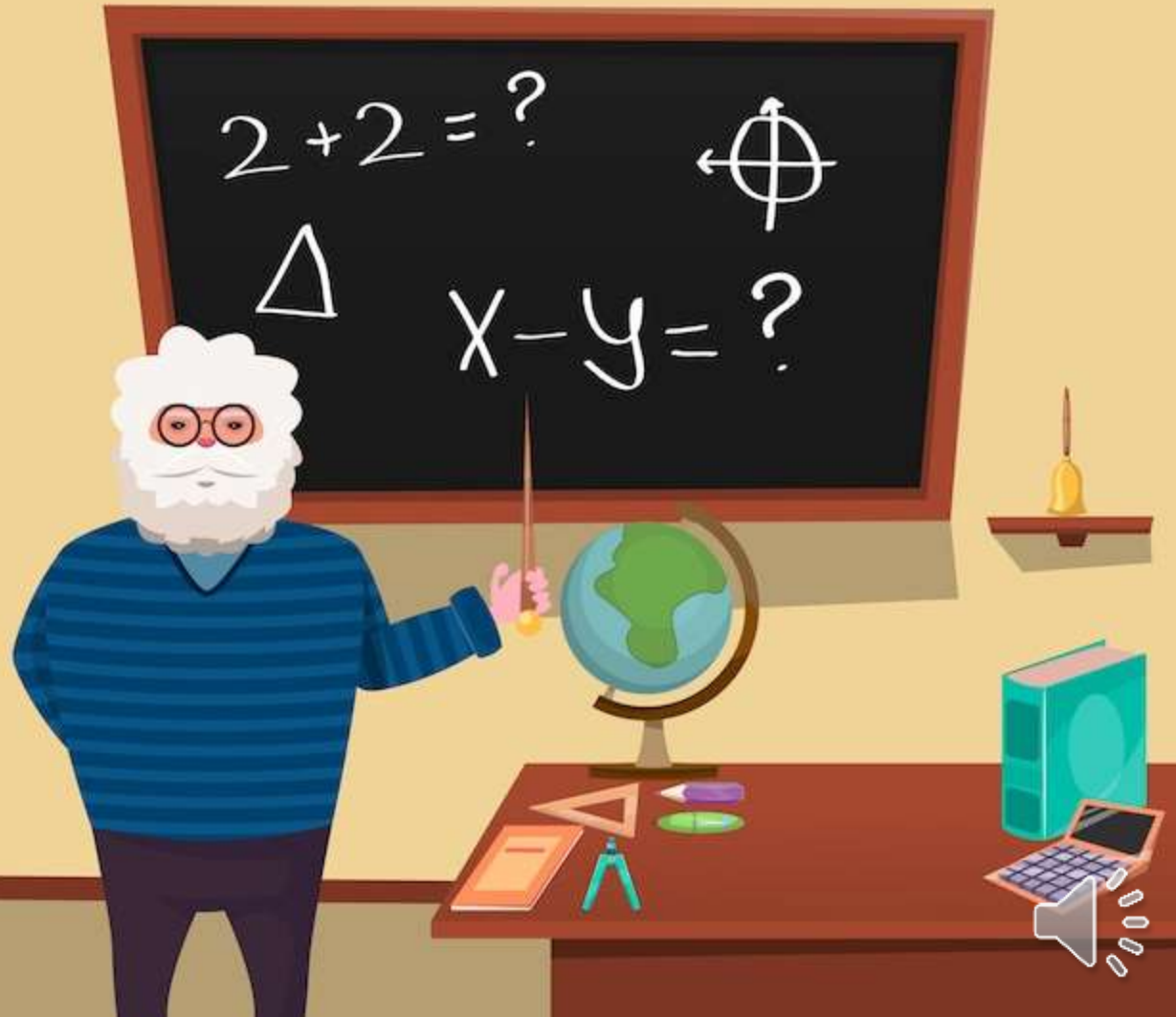
Algebraic Expressions

Part 5



Recall

Expand the following :

1) $4(x + 5) =$ $4x + 20$

2) $3(y + 9) =$ $3y + 27$

3) $5(a - 2) =$ $5a - 10$

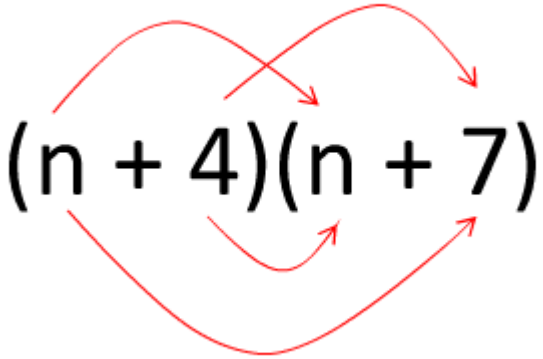
4) $11(r - 4) =$ $11r - 44$

5) $a(b + 3) =$ $ab + 3a$

6) $n(n + 5) =$ $n^2 + 5n$

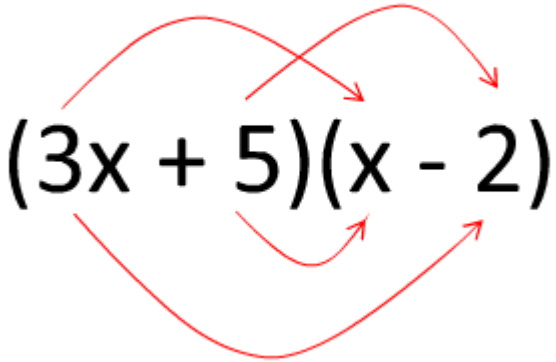
7) $p(8 - p) =$ $8p - p^2$

Expand



$= n^2 + 7n + 4n + 28$

$= n^2 + 11n + 28$



$= 3x^2 - 6x + 5x - 10$

$= 3x^2 - x - 10$

Expand

$(a + 8)(a + 7) = a^2 + 15a + 56$

$(b + 2)(3 - b) = -b^2 + b + 6$

$(c + 7)(c - 3) = c^2 + 4c - 21$

$(2d + 5)(-d + 1) = -2d^2 - 3d - 5$

$(3e + 11)(4e - 11) = 12e^2 + 11e - 121$

$(3f - 7)(5f - 7) = 15f^2 - 56f + 49$

$(g + 8)(-2g - 1) = -2g^2 - 17g - 8$

Remarkable Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Expand the following expressions using the remarkable identities

$a)\ (x+3)^2 \qquad b)(x-3)^2 \qquad c)(x+5)(x-5)$

Solution:

$$a)\ (x+3)^2 = x^2 + 2\times x\times 3 + 3^2 = x^2 + 6x + 9$$

$$b)\ (x-3)^2 = x^2 - 2\times x\times 3 + 3^2 = x^2 - 6x + 9$$

$$c)\ (x+5)(x-5) = x^2 - 5^2 = x^2 - 25$$

Expand the following expressions using the remarkable identities

$a)\ (2x+5)^2 \qquad b)(3x-2)^2 \qquad c)(4x+7)(4x-7)$

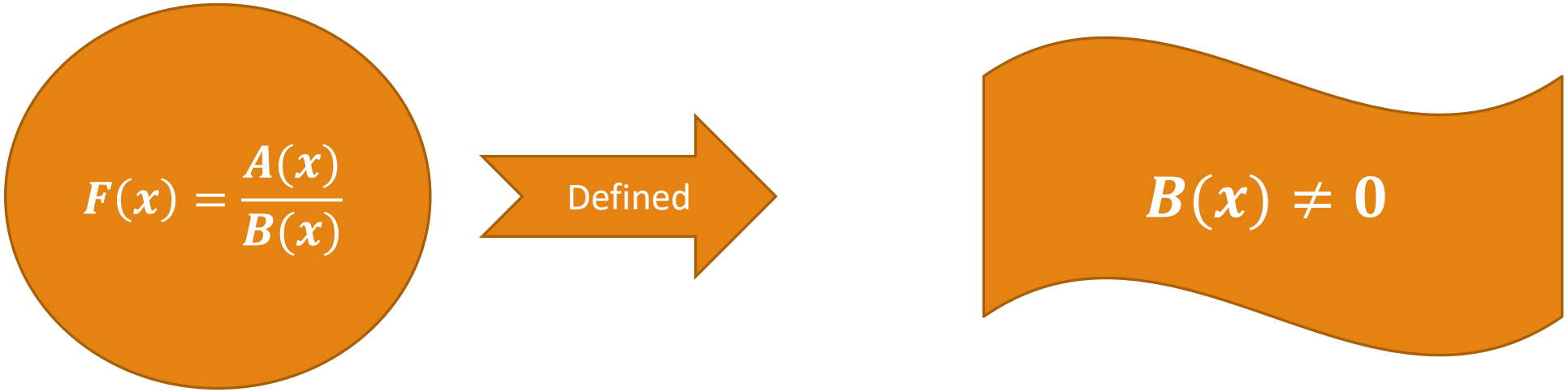
Solution:

$$a)\ (2x+5)^2 = (2x)^2 + 2\times 2x\times 5 + 5^2 = 4x^2 + 20x + 25$$

$$b)\ (3x-2)^2 = (3x)^2 - 2\times 3x\times 2 + 2^2 = 9x^2 - 12x + 4$$

$$c)\ (4x+7)(4x-7) = (4x)^2 - 7^2 = 16x^2 - 49$$

When is the fractional expression defined?



Find the values of x for which the following fractional expressions are defined.

$$1)P(x)=\frac{3x^2+x+1}{5x-20} \qquad 2)Q(x)=\frac{2}{4x^2-1} \qquad 3)R(x)=\frac{2x-3}{9x^2+6x+1}$$

Solution:

$$1)P(x)=\frac{3x^2+x+1}{5x-20}$$

The expression $P(x)$ is defined when: $5x-20\neq 0$

$$5x\neq 20$$

$$x\neq \frac{20}{5}$$

$$x\neq 4$$

Thus the expression $P(x)$ is defined for every real number except 4.

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$$2)Q(x)=\frac{2}{4x^2-1}$$

The expression $Q(x)$ is defined when: $4x^2-1\neq 0$

$$(2x-1)(2x+1)\neq 0$$

$$2x-1\neq 0 \quad \text{and} \quad 2x+1\neq 0$$

$$2x\neq 1 \quad \text{and} \quad 2x\neq -1$$

$$x\neq \frac{1}{2} \quad \text{and} \quad x\neq -\frac{1}{2}$$

Thus the expression Q(x) is defined for every real number except $\frac{1}{2}$ and $-\frac{1}{2}$.

Find the values of x for which the following fractional expressions are defined.

$$1)P(x)=\frac{3x^2+x+1}{5x-20} \qquad 2)Q(x)=\frac{2}{4x^2-1} \qquad 3)R(x)=\frac{2x-3}{9x^2+6x+1}$$

Solution:

$$3)R(x)=\frac{2x-3}{9x^2+6x+1}$$

The expression $R(x)$ is defined when: $9x^2+6x+1\neq 0$

$$(3x+1)^2\neq 0$$
$$3x+1\neq 0$$
$$3x\neq -1$$
$$x\neq -\frac{1}{3}$$

Thus the expression $R(x)$ is defined for every real number except $-\frac{1}{3}$.

Simplify

Simplify the following **fractional** expressions:.

1) $P(x) = \dfrac{x^2 - 1}{5x - 5}$ 2) $Q(x) = \dfrac{x^2 - 9}{x^2 + 6x + 9}$ 3) $R(x) = \dfrac{x^3 - 2x^2}{x^2 - 4}$

Solution:

1) $P(x) = \dfrac{x^2 - 1}{5x - 5} = \dfrac{(x - 1)(x + 1)}{5(x - 1)}$

$P(x)$ is defined when: $x - 1 \neq 0$ Thus if $x \neq 1$

$$= \frac{x + 1}{5}$$

Simplify

Simplify the following **fractional** expressions:.

1) $P(x) = \dfrac{x^2 - 1}{5x - 5}$ 2) $Q(x) = \dfrac{x^2 - 9}{x^2 + 6x + 9}$ 3) $R(x) = \dfrac{x^3 - 2x^2}{x^2 - 4}$

2) $Q(x) = \dfrac{x^2 - 9}{x^2 + 6x + 9} = \dfrac{(x-3)(x+3)}{(x+3)^2}$

$Q(x)$ is defined when: $(x+3)^2 \neq 0$ Thus if $x \neq -3$

$$= \frac{x-3}{x+3}$$

Simplify

Simplify the following **fractional** expressions:.

1) $P(x) = \dfrac{x^2 - 1}{5x - 5}$ 2) $Q(x) = \dfrac{x^2 - 9}{x^2 + 6x + 9}$ 3) $R(x) = \dfrac{x^3 - 2x^2}{x^2 - 4}$

3) $R(x) = \dfrac{x^3 - 2x^2}{x^2 - 4} = \dfrac{x^2(x-2)}{(x-2)(x+2)}$

$R(x)$ is defined when $(x-2)(x+2) \neq 0$

Thus if $x \neq 2$ and $x \neq -2$

$$= \frac{x^2}{x+2}$$

Solving Equations

Given $A(x) = (2x - 1)(3x + 7)$

Solve :

- $A(x) = 0$
- $A(x) = (2x - 1)$
- $A(x) = 3(1 - 2x)$

Solving Equations

Given $A(x) = (2x - 1)(3x + 7)$

$$\underline{A(x) = 0}$$

- $(2x - 1)(3x + 7) = 0$
- $2x - 1 = 0 \quad or \quad 3x + 7 = 0$
- $x = \frac{1}{2} \quad or \quad x = \frac{-7}{3}$

Solving Equations

Given $A(x) = (2x - 1)(3x + 7)$

$$\underline{A(x) = (2x - 1)}$$

- $(2x - 1)(3x + 7) = 2x - 1$
- $\underline{(2x - 1)}(3x + 7) - \underline{(2x - 1)} = 0$
- $(2x - 1)(3x + 6) = 0$
- $2x - 1 = 0 \quad or \quad 3x + 6 = 0$
- $x = \frac{1}{2} \quad or \quad x = -2$

Solving Equations

Given $A(x) = (2x - 1)(3x + 7)$

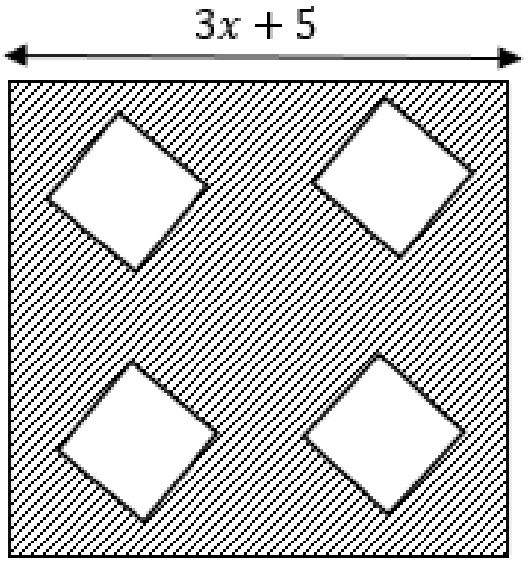
$\underline{A(x) = 3(1 - 2x)}$

- $(2x - 1)(3x + 7) = 3(1 - 2x)$
- $(2x - 1)(3x + 7) - 3(1 - 2x) = 0$
- $\underline{(2x - 1)}(3x + 7) + 3\underline{(2x - 1)} = 0$
- $(2x - 1)(3x + 10) = 0$
- $2x - 1 = 0 \quad or \quad 3x + 10 = 0$
- $x = \frac{1}{2} \quad or \quad x = -\frac{10}{3}$

Solved Exercise

Given the following expression $A(x) = (3x + 5)^2 - 4(x + 1)^2$

A big square of side $3x + 5$ contains four identical squares of side $x + 1$ each $(x > -1)$



1) Factorize $A(x)$.
2) Expand and reduce $A(x)$.
3) a) Solve A(x)= 21

   b) Deduce the value of x so that the area of the shaded part is 21 ul^2
4) Consider the following expression : $B(x) = \frac{A(x)}{-10x^2+11x+35+(-5x-7)(4-3x)}$

   a) Expand and reduce $(5x + 7)(5 - 2x)$

   b) Deduce the simplified form of $B(x)$

   c) Find $B\left(\frac{-1}{3}\right)$

Parts	Answers
1.	$A(x) = (3x+5)^2 - 4(x+1)^2 = (3x+5)^2 - [2(x+1)]^2$ $= (3x+5+2x+2)(3x+5-2x-2) = \boxed{(5x+7)(x+3) = A(x)}$
2.	$A(x) = (5x+7)(x+3) = (5x^2+15x+7x+21) = \boxed{5x^2+22x+21 = A(x)}$
3.a	$A(x) = 5x^2+22x+21 = 21$ $5x^2+22x = 0$ $x(5x+22) = 0$ then $\boxed{x=0}$ or $5x+22=0$, $\boxed{x = -\frac{22}{5}}$
3.b	$A_{shaded\ part} = A_{big\ square} - 4A_{small\ square} = (3x+5)^2 - 4(x+1)^2 = A(x)$ $A_{shaded\ part}=21 \iff A(x) = 21$ from part 3.a) then $\boxed{x = 0 > -1}$ accepted or $x = \frac{-22}{5} < -1$ rejected
4.a	$(5x+7)(5-2x) = (25x - 10x^2 + 35 - 14x) = -10x^2 + 11x + 35$
4.b	$B(x) = \frac{A(x)}{-10x^2+11x+35+(-5x-7)(4-3x)} = \frac{(5x+7)(x+3)}{(5x+7)(5-2x)-(5x+7)(4-3x)} = \frac{(5x+7)(x+3)}{(5x+7)(5-2x-4+3x)}$ $= \frac{(5x+7)(x+3)}{(5x+7)(x+1)}$ $(5x+7)(x+1) \neq 0$, $5x+7 \neq 0$, & $x+1 \neq 0$, then $\boxed{x \neq \frac{-7}{5} \ \& \ x \neq -1}$ $\boxed{B(x) = \frac{x+3}{x+1}}$
4.c	$B\left(\frac{-1}{3}\right) = \frac{\frac{-1}{3}+3}{\frac{-1}{3}+1} = \frac{\frac{-1}{3}+\frac{9}{3}}{\frac{-1}{3}+\frac{3}{3}} = \frac{\frac{8}{3}}{\frac{2}{3}} = \frac{8}{3} \times \frac{3}{2} = \frac{8}{2} = 4$ then $\boxed{B\left(\frac{-1}{3}\right) = 4}$